$$\Rightarrow \sum_{k=0}^{n} (-1)^k \cdot \int_0^1 t^{2k}\, dt - I = (-1)^n \cdot U_n$$

$$\Rightarrow \sum_{k=0}^{n} (-1)^k \cdot \left[\frac{t^{2k+1}}{2k+1}\right]_0^1 - I = (-1)^n \cdot U_n$$

$$\Rightarrow \sum_{k=0}^{n} (-1)^k \cdot \frac{1}{2k+1} - I = (-1)^n U_n$$

$$\Rightarrow \boxed{V_n - I = (-1)^n \cdot U_n}$$

c-)Montrer que $|v_n - I| \leq \frac{1}{2n+1}$ puis déduire la laimite de la suite $(v_n)_n$

on $V_n - I = (-1)^n U_n$

$\Rightarrow |V_n - I| = |(-1)^n U_n|$

$= |U_n| = U_n \leq \frac{1}{2n+1}$

donc $|V_n - I| \leq \frac{1}{2n+1}$

on $\lim_{n \to +\infty} \frac{1}{2n+1} = 0$